ABSTRACTS OF ARTICLES DEPOSITED AT VINITI*

HEAT TRANSFER IN FREE MOTION IN HORIZONTAL LAYERS HEATED FROM BELOW

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UDC 536.248.2

It is shown that the results of existing theoretical solutions for heat transfer in plane horizontal layers heated from below differ considerably from one another. The need for a wider generalization is demonstrated by analyzing new experimental work and existing generalizations on heat transfer through a layer after loss of stability.

A total of 605 experimental points taken from 12 publications for the regions of heat conduction, convection, and laminar and turbulent motion of a fluid in the layer are processed by the method of least squares. A justification for the choice of the limits of generalizations for each zone is given. For a 95% confidence coefficient and a mean-square deviation of 9.2%, the correlation between the processed material is represented by zones with the following four equations:

$$\mathrm{Nu} = 1;\ \mathrm{Ra} < 1740; \tag{1}$$

$$\mathrm{Nu} = 1 + 1{,}44\left(1 - \frac{1740}{\mathrm{Ra}}\right);\ 1740 < \mathrm{Ra} < 4 \cdot 10^3; \tag{2}$$

$$\mathrm{Nu} = 0{,}24\,\mathrm{Ra}^{0{,}25};\ 4 \cdot 10^3 < \mathrm{Ra} < 3 \cdot 10^6; \tag{3}$$

$$\mathrm{Nu} = 0{,}115\,\mathrm{Ra}^{0{,}3};\ 3 \cdot 10^6 < \mathrm{Ra} < 10^{10}; \tag{4}$$

The results obtained are compared with theoretical solutions and are found to agree only qualitatively. It is shown that the widely used Mikheev generalization, obtained for layers of different configurations cannot be recommended even for approximate calculations of heat transfer through a plane layer heated from below. Equations (1), (2), and (3) approximate the experimental data most closely in the range of variation of Prandtl numbers $0.7 < \mathrm{Pr} < 11760$, while Eq. (4) approximates the data in the range $0.7 < \mathrm{Pr} < 6$. These relations are reliable, convenient, and fairly accurate for practical calculations.

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HYDRODYNAMICS OF THE VORTEX CHAMBER

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The laminar motion of a viscous incompressible liquid in a vortex chamber with a tangential component of the velocity depending only on the radius is considered. In this formulation the Navier–Stokes equations reduce to a system of ordinary differential equations. When the boundary conditions are assigned the solution can be divided into two regions – a central region and a peripheral region. The boundary-value problem is solved by the ranging method for the differential equations. On the basis of the numerical solutions approximate analytical solutions are obtained which depend on a single constant m. It is shown that the constant is determined by the twist in the vortex chamber. A whole class of possible solutions is analyzed and m is found for zero twist. The values of m for different twists can be uniquely determined by comparing the calculated velocity and pressure fields with the measured fields [1]. Figure 1 shows calculated fields of the radial component of the velocity u, the tangential component v, and the axial component w, the pressure drop Δp for m = 1.657 and a radius of the exit opening R_1 = 1.5 cm, and also the streamlines. It is seen from the figure that: 1) The velocity and pressure fields obtained are qualitatively similar to the experimental fields and represent all the characteristic features of the motion of a gas in vortex chambers, namely, the presence of radial and axial flows in opposite directions; 2) that a maximum of the tangential velocity is obtained inside the opening. On the figure we can also note the characteristic shape of the curve showing the variation of the pressure along the radius.

Analytical expressions are given for the velocity and pressure fields in the vortex chamber as a function of the constant m. These expressions also describe the special case of untwisted flow, for example, the flow in a porous tube with a single impenetrable end, for which the value of the constant is determined theoretically. The results of the calculation are compared with published data for a porous tube and agree with them quantitatively.

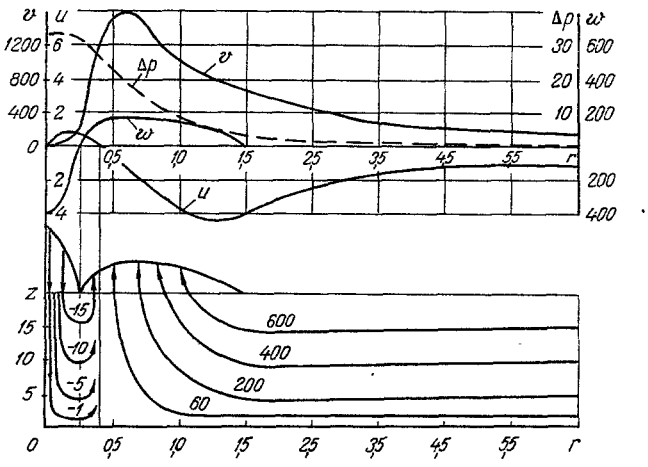

Fig. 1. Velocity and pressure fields and stream lines in a vortex chamber (m = 1.657); r and z are in centimeters; u, v, and w are in cm/sec; and Δp is in mm H_2O.

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HEAT TRANSFER IN PLANE FREE JETS OF A VISCOUS INCOMPRESSIBLE FLUID

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UDC 532.517

The main results of the paper can be summarized as follows. The second and third terms of the asymptotic expansions of the temperature in problems of the development of free plane laminar and turbulent jets of a viscous incompressible fluid with a "symmetrical" distribution of the temperature in the surrounding space are obtained in finite form. The solution is obtained using the equations of the boundary layer. In the problem of the development of a turbulent jet the semiempirical Prandtl theory of turbulence and the Fourier hypothesis for heat conduction are used. The results of experimental investigations of the distribution of the concentration of the gaseous mixture in plane free turbulent jets of air with different initial nonuniformities of the outflow velocity from the slit are presented. The expression obtained for the temperature distribution is used to describe the distribution of the concentration of the gaseous impurity in the free turbulent jets of air being investigated (with corresponding replacement of the Prandtl number by the Schmidt number) in the transient parts. The effect of the shape of the initial outflow velocity profile from the slit on the temperature distribution in a plane free laminar jet with an "asymmetrical" temperature distribution in the space surrounding the jet is investigated.

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AN OPTIMAL CRYOGENIC HEAT-INSULATION SYSTEM

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UDC 621.59.04−76−03.

The efficiency of cryogenic heat-insulating envelopes can be increased considerably if the heat is withdrawn at intermediate temperature levels, for example, using cooled screens. The idea of relative energy cost representing the ratio of the cost of the power required to compensate the heat inflow through the insulation and the cooling of the screens to the cost of the power without intermediate cooling is proposed as a characteristic of the efficiency of intermediate cooling. It is shown that for each insulation, because of the nature of the temperature dependence of the thermal conductivity, there are optimum temperatures of the screens and their position with respect to the insulation thickness. Calculations show that the use of cooled screens is the more effective, the greater the temperature dependence of the thermal conductivity of the insulation and the greater the ratio of the temperatures of the hot and cold surfaces. Cooled screens can be employed most effectively for cryogenic devices having a temperature below 10°K using screen-vacuum heat insulation as a heat shield. The results of a calculation of the relative energy costs E, the optimum screen temperature T, and their optimum relative distance from the cold surface x for the case of screen-vacuum heat insulation with boundary temperatures of 4.2-300°K are presented in Table 1. The energy costs are calculated assuming ideal Carnot cycles.

It is shown that nonuniformity of the optimum position of the screens – increasing the frequency of their position in the region of the cold surface – is the greater, the greater the dependence of the thermal conductivity on the temperature. It is characteristic that for any form of this dependence the use of a large number of cooled screens (in practice, greater than three) is not very effective. In the majority of cases it is sufficient to introduce a single cooled screen into the insulation. Calculation has shown that consideration of the actual cooling coefficient hardly changes the optimum temperature of the screens and their optimum distribution, but considerably reduces the relative energy costs, i.e., the use of cooled screens is actual systems is more effective than would follow from the table.

TABLE 1. Optimum Parameters of Systems with Cooled Screens

Parameter	No. of screens			
	1	2	3	5
E	0,071	0,04	0,033	0,028
x	0,25	0,1 0,39	0,055 0,201 0,5	0,019 0,072 0,175 0,35 0,62
T, °K	79	45 142	38 96 183	20 49 92 147 217

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EXPERIMENTAL DETERMINATION OF THE EFFECT OF DIMERIZATION OF MOLECULES OF BINARY MIXTURES OF INERT GASES ON THERMAL-DIFFUSION SEPARATION

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UDC 533.735

According to [1, 2, 3], at low temperatures double molecule-dimers are formed, the number of which increases as the pressure of the mixture increases, as the temperature is reduced, and as the content of heavier gas in the mixture increases. Dimerization of the mixture leads to anomalies in the temperature and concentration dependences of the thermal-diffusion constant (α_T) [2, 3].

The purpose of this work was to make an experimental determination of the effect of dimerization on thermal-diffusion separation.

Using the two-vessel method [4], we investigated the temperature and concentration dependences of the thermal-diffusion constant of Kr–Ar, Xe–Ar, and Xe–Kr mixtures in the temperature range T = 168-271°K, where

$$\overline{T} = T_u T_l \ln \frac{T_u}{T_l} (T_u - T_l)^{-1}. \quad (1)$$

The experiments were made at the two values of the start-up pressure of the mixture in the apparatus – 380 mm Hg and 228 mm Hg. The results show a monotonic change in α_T with temperature for all the concentrations of the mixtures investigated provided that the ratio P_S/P_0 (where P_S is the saturation vapor pressure of the heavy component in the mixture at the given temperature of the lower vessel, and P_0 is the partial pressure of the heavy component at the given temperature of the lower vessel) is greater than 2.05 for the Kr–Ar mixture and greater than 1.8 for the Xe–Ar and Xe–Kr mixtures.

When the temperature in the lower vessel is reduced so that the ratio P_S/P_0 becomes less than 2.05 and 1.8 for the corresponding mixtures, a minimum is found from the temperature dependence of α_T at all concentrations of these mixtures. Experiments carried out with pure gas (Kr) showed that when the temperature in the lower vessel is reduced to values corresponding to $P_S/P_0 < 2.05$, there is a sharp increase in the thermal conductivity of the gas. This effect cannot be explained by dimerization, since it does not lead to an increase in the thermal conductivity of the gas. On the other hand, since the temperature range between the linear phase transition and the temperature corresponding to the ratio $P_S/P_0 = 2.05$ is of the order of 4-5°, the increase in the thermal conductivity may be due to the fact that the state of the system approaches the phase transition. In our opinion, this is the explanation for the anomalies in the temperature dependences of α_T observed when $P_S/P_0 = 2.05$ for Kr–Ar mixtures and for $P_S/P_0 = 1.8$ for Xe–Ar and Xe–Kr mixtures.

We carried out an analysis of the data in the literature on the thermal-diffusion constant for the gaseous mixtures investigated at low temperatures. All the results considered were in the temperature range for which the ratio $P_s/P_0 > 3$, and, with the exception of [3], not one of them mentions the effect of dimerization on the thermal-diffusion separation of gaseous mixtures.

We calculated the values of α_T both for a binary mixture using the Lennard–Jones potential and for a ternary mixture using the method described in [5]. The concentration of the dimers in the mixtures investigated was determined as in [1]. The theoretical calculations do not reveal any appreciable effect of dimerization of the mixture on thermal-diffusion separation (the deviation of α_T^{ter} from α_T^{bin} for the corresponding temperatures is within the limits of experimental error).

NOTATION

T_u is the temperature of the upper vessel;
T_l is the temperature of the lower vessel;
α_T^{bin} is the thermal-diffusion constant of a binary mixture;
α_T^{ter} is the thermal-diffusion constant of a ternary mixture.

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DETERMINATION OF THE NONSTEADY-STATE HEAT-TRANSFER COEFFICIENT BY THE METHOD OF INTEGRAL CHARACTERISTICS

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UDC 536.24.02

Using the method described in [1], the problem of extrapolation of the thermal information in terms of a physical medium with properties which vary with respect to the spatial coordinate is solved. The nonsteady-state heat-transfer coefficient $\alpha(t)$ is found from the boundary-value problem

$$\frac{\partial u_i(x, t)}{\partial t} = a_i \frac{\partial^2 u_i(x, t)}{\partial x^2}, \quad t > 0, \quad l_{i-1} < x < l_i, \quad l_0 = 0, \quad l_n = l, \tag{1}$$

$$u_i(x, 0) = 0, \quad i = 1, 2, \ldots, n, \quad u_1(0, t) = g(t), \quad u_k(l_k, t) = h(t), \quad 1 \leqslant k \leqslant n, \tag{2}$$

$$u_i(l_i, t) = u_{i+1}(l_i, t), \quad \lambda_i \frac{\partial u_i(l_i, t)}{\partial x} = \lambda_{i+1} \frac{\partial u_{i+1}(l_i, t)}{\partial x}, \quad i = 1, 2, \ldots, n-1,$$

$$-\lambda_n \frac{\partial u_n(l_n, t)}{\partial x} = q(t) = \alpha(t)[u_n(l_n, t) - u_{cp}(t)],$$

where the thermal conductivity λ_i, the thermal diffusivity a_i, and the geometrical dimensions l_i are known and constant; the temperatures g(t), h(t), and $u_{av}(t)$ are known for all $t > 0$, and the subscript k in (2) takes only one fixed value. The procedure for calculating q(t) is as follows.

1. From the solution of problem (1), (2) we determine the heat flux

$$q_k(t) = -\lambda_k \frac{\partial u_k(l_k, t)}{\partial x}.$$

2. The functions Q(t) and H(t) are found from the integral equations of the first kind

$$q_k(t)=\int_0^t \psi\left(\sum_{i=k+1}^{n} L_i,\ t-\tau\right) Q(\tau)\,d\tau,\quad h(t)=\int_0^t \psi\left(\sum_{i=k+0} L_i,\ t-\tau\right) H(\tau)\,d\tau, \tag{3}$$

$$\psi(y,\ t)=\frac{y}{2t\sqrt{\pi t}}\exp\left(-\frac{y^2}{4t}\right),\quad L_i=\frac{l_i-l_{i-1}}{\sqrt{a_i}}.$$

The solution of Eqs. (3) can be found, for example, by the regularization method described for such equations in [2].

3. $q(t) = Q_n(t)$ and $u_n(l, t) = H_n(t)$, where $Q_n(t)$ and $H_n(t)$ are obtained by integrating Q(t) and H(t) with use of the recurrent relations given in the text.

4. The quantity

$$\alpha(t)=\frac{q(t)}{u_n(l,\ t)-u_{av}(t)}$$

is then calculated.

For the case when the thermophysical coefficients λ_i and a_i, $i = k + 1, \ldots, n$, are unknown, the problem of extrapolation of the thermal information reduces to solving a system of nonlinear algebraic equations containing the integral characteristics of the temperature.

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ANALYTICAL RELATIONS FOR CALCULATING THE STEADY-STATE TEMPERATURE FIELD AND VARIABLE BOUNDARIES IN A HEAT EXCHANGER

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UDC 536.27

An adequate mathematical description of a heat exchanger requires the consideration and solution of a nonlinear boundary-value problem for a system of differential equations in partial derivatives. The theoretical form of such a heat exchanger is an equivalent system of the "tube in a tube" type and in the second arrangement contains an economizer, an evaporator, and a vapor-reheating section with movable phase-transition boundaries. The difficulty in solving this problem is due to the time-varying boundaries of the evaporator seciion $z_1(\tau)$ and to the formation of reheated vapor $z_2(\tau)$, defined implicitly by the initial system of equations and also by the system of initial and boundary conditions. The variability of the boundaries is due to displacement of the latter as a function of the power entering and leaving the heat exchanger.

When simulating the nonsteady-state modes of operation of a heat exchanger on a computer taking into account the distribution of the parameters, it is necessary to know the steady-state temperature distribution for the initial conditions. In this paper, analytical relations for calculating the steady-state temperature field of the economizer, evaporator, and vapor-reheating sections and equations for the variable boundaries z_1 and z_2 of the economizer and evaporator sections are obtained both for a two-zone and a three-zone heat exchanger.

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ION TRANSFER IN A POROUS MEMBRANE WITH FIXED CHARGE

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The transfer of ions in "thick" porous membranes having fixed charge is considered. The potential distribution and charge in the pores is determined by solving the Poisson – Boltzmann equation

$$\frac{d^2\psi}{dr^2}+\frac{1}{r}\frac{d\psi}{dr}=-\frac{4\pi F}{\varepsilon}\left(c_{+0}e^{-F\psi/RT}-c_{-0}e^{F\psi/RT}\right). \tag{1}$$

The boundary condition for Eq. (1) at 0 can be written [Eq. (1) is singular at 0]

$$\psi(0)=0. \tag{2}$$

The approximate solution of this equation enables one to calculate the activity of the positive and negative ions in the membrane phase. Thus, we have

$$a_-=\gamma_{-0}c_{-0}, \tag{3}$$

$$a_+=\gamma_{+0}c_{+0}=\gamma_{+0}\left[c_{-0}+c^*_{+0}f\left(\frac{c_{-0}}{c^*_{+0}}\right)\right], \tag{4}$$

where

$$f\left(\frac{c_{-0}}{c^*_{+0}}\right)\equiv f(x)=\frac{1-\dfrac{(p-1+b^2/2)}{A}x^{1/4}+bx^{1/2}+x^{3/4}\dfrac{[b(p-1+b^2/2)]}{A}+\left(\dfrac{\dfrac{b^2}{2}-1+p}{1}\right)x}{1+\dfrac{(p-1+b^2/2)x^{1/4}}{A}}e^{-bx^{1/2}}; \tag{5}$$

$$b\equiv 4\frac{\lambda^{1/2}}{(2+\lambda)^{1/2}};\quad A\equiv\sqrt{8\pi}\lambda^{3/4}(2+\lambda)^{1/4};\quad p=\frac{\lambda+2}{3\lambda+2}.$$

The potential obtained by solving Eq. (1) and also the activity calculated from (3), (4), and (5) are used in the ion-transfer equations in the membrane to determine the concentrational dependence of the diffusion potential difference and electrical resistance. Good agreement with experiment is observed for certain natural membranes.

NOTATION

ψ is the potential;
F is the Faraday number;
R is the gas constant;
c_{+0} and c_{-0} are the concentrations of counterions and co-ions, respectively, on the axis of a pore;
γ_{+0}, γ_{-0} are the activity coefficients;
a_+, a_- are the activities of the positive and negative ions, respectively;
a is the radius of a pore;
$\lambda\equiv -2\pi\sigma Fa/\varepsilon RT$;
σ is the surface charge density in a pore;
ε is the dielectric constant of the aqueous medium in the membrane.

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TEMPERATURE FIELD OF A RUBBER – METAL COMPONENT DURING CONVECTIVE VULCANIZATION OF ITS ELASTOMER COATING

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The vulcanization of a layer of elastomer deposited on a metal substrate occurs at relatively high temperatures (200-250°C) and is accompanied by a considerable time-varying temperature gradient over the thickness of both the substrate and the vulcanized elastomer coating. Hence, the temperature field of the metal and the elastomer enables one to predict the quality of the rubber–metal component, to estimate how the vulcanization is progressing with time, and thus to choose the optimum vulcanization procedure.

An "infinite" two-layer plate, one of the layers of which is the metal substrate, the other being an adhesive layer between the metal and the elastomer and several joined layers of elastomer, is chosen as the model of convective vulcanization of the rubber–metal component, i.e., the problem of the analytical investigation of the temperature field of a rubber–metal component is formulated as a one-dimensional problem – the temperature of the metal and the elastomer only varies along the thickness of the plate and during the vulcanization process. In addition, the smallness of the integral heat effect of the process compared with the heat flow supplied from the vulcanizing medium enables one to introduce the assumption that there are no heat sources in the model and to describe the heat transfer in the elastomer and metal using the same equations.

The solution of the problem is found by an operational method. A series of experiments in which the temperature field of the rubber–metal component was measured showed satisfactory agreement between the theoretical and experimental values of the temperature of the metal and elastomer. It should be noted that the method of calculating the temperature field can be used for any two-layer component which can be reduced to an "infinite" plate arrangement and which is subjected to convective thermal processing, assuming that there is no internal heat dissipation.

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INVESTIGATION OF THE KINETICS OF THE DIELECTRIC DRYING OF LEATHER

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Specimens of dimensions 130 mm × 215 mm of chrome leather for the uppers of footwear with an initial moisture content W_0 = 96-154% were packed face downward on the electrodes of a rod-type capacitor and dried at a current frequency f = 63.7-65.1 MHz and a voltage U_a = 1.875-2.5 kV. The available oscillatory power of the equipment was 0.6 kV. The first period of drying extended down to $W_{cr,1} \approx 60$-65%, which was practically independent of W_0, the thickness δ of the specimens, and the electric field strength E in them, and is considerably lower than $W_{cr,1}$ for convective drying. The relative length of this drying period is increased compared with convective drying, where the values of $W_{cr,1}$ closer to the lower of these limits, correspond to higher values of E.

In the first period the rate of drying N and the drying intensity j, referred to the total geometrical area of the specimen, depend on S and E, decreasing as δ increases. The variation of W_0 from 103% to 154% does not affect N and j to any appreciable extent. When δ is reduced by a factor of 2, N increases by a factor of 2-2.5; j varies from 3.36 kg/m^{2}h for U_a = 1.875 kV, W_0 = 135%, and δ = 1.75 mm to 10.12 kg/m^{2}h for U_a = 2.46 kV, W_0 = 140%, and $\delta \approx 0.8$. For fixed δ and W_0, N and j increase when E increases due to the increase in U_a. The dependence of N and j on U_a^2 is nonlinear.

The drying curves in the second period can be approximated by an empirical formula of the form

$$W = W_{cr,1} - \frac{\tau^*}{n + \varkappa\tau^*}, \quad (1)$$

where τ^* is the time measured from the beginning of the period; n = 1/N; and $\varkappa \approx 0.0245$ and is independent of the mode of drying. The difference between the theoretical and experimental drying curves does not exceed 14% in the moisture-content region close to equilibrium.

In the case considered the well-known hypothesis put forward by V. V. Krasnikov with regard to the equation $(N_\tau)_W$ = const is confirmed. Shrinkage of the leather is observed as the process proceeds. When the specimens are dried from W_0 = 115% to W = 20%; their area F decreases by approximately 10% (particularly when W < 40%). The experimental data can be approximated by relations of the form

$$F_W = F_0(1 + \beta'_F W), \; W \leqslant 35\%; \quad (2)$$

$$F_W = F_0[1.24 + \beta''_F(W - 0.4)], \; 40 \leqslant W \leqslant 160\%, \quad (3)$$

where $\beta'_F \approx 0.6$ and $\beta''_F \approx 0.42$ and are independent of the form and thickness of the leather, the mode of drying, and on whether the specimens are first dried or are dried after secondary moistening.

The drying of a nonuniformly moistened skin is accompanied by compensation of the moisture content with increasing efficiency when $W_1 > W_{cr,1} > W_2$. If $W_1 > W_2 > W_{cr,1}$, then up to the instant when the equation $W_2 = W_{cr,1}$ holds, the nonuniformity of the moisture content increases. After this instant the rate of increase of the nonuniformity slows down, becomes equal to zero, and the moisture contents of separate parts of the skin then become equalized.

Results of the investigation confirm that the use of dielectric drying instead of vacuum-contact drying of leather in the unfixed state is promising. It is best to use dielectric heating in the two-state finishing of leather for conditioning it before stretching after vacuum-contact drying.

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OVERALL TEMPERATURE FIELDS IN HOLLOW SYMMETRICAL UNIFORM BODIES

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UDC 517.946

For highly intense nonstationary modes the true temperature field in an isotropic uniform hollow symmetrical body $D = [0, t_1] \times \Omega = \{(t, r), 0 \le t \le t_1, R_1 \le r \le R_2\}$ describes the scalar quantity T which is the solution of the generalized (hyperbolic) equation of heat conduction [1]

$$b_0^2 \frac{\partial^2 T}{\partial t^2} + b_1^2 \frac{\partial T}{\partial t} - a^2 \left(\frac{\partial^2 T}{\partial r^2} + \frac{2\nu + 1}{r} \frac{\partial T}{\partial r} \right) = f_1(t, r) \tag{1}$$

for the initial and boundary conditions

$$T|_{t=0} = f_2(r), \quad \frac{\partial T}{\partial t}\Big|_{t=0} = f_3(r), \tag{2}$$

$$B_j[T]|_{r=R_j} \equiv \left(h_{j1} \frac{\partial}{\partial r} + h_{j2} \frac{\partial}{\partial t} + h_{j3} \right) T\Big|_{r=R_j} = (-1)^{j+1} \omega_j(t), \quad j = 1, 2. \tag{3}$$

Here $2\nu + 1 \geqslant 0$, $a/b_0 = w_r$, $h_{11} = h_{21} = \lambda_T$, $h_{12} = \alpha_1\beta_1\beta_2\tau_r$, $h_{13} = \alpha_1\beta_1$, $h_{22} = -\alpha_2\beta_3\beta_4\tau_r$, $h_{23} = -\alpha_2\beta_3$; β_k $(k = \overline{1, 4})$ are arbitrary parameters which enable solutions to be obtained for any of the first three kinds of boundary conditions assigned on any of the surfaces $r = R_j$ $(j = 1, 2)$ or on both simultaneously; b_0 and b_1 are arbitrary nonnegative parameters which enable one to obtain both a pure wave $(b_1 \to 0)$, and the ordinary (parabolic) temperature fields $(b_0 \to 0, b_1 = 1)$; the functions ω_j are given, respectively, by

$$\omega_1(t) = \alpha_1 \left(1 + \tau_r\beta_2 \frac{\partial}{\partial t} \right) f_4(t), \quad \omega_2(t) = \alpha_2 \left(1 + \tau_r\beta_4 \frac{\partial}{\partial t} \right) f_5(t).$$

The remaining quantities have their usual meanings [1]. When $\nu = 0$, we have the case of axial symmetry, and when $\nu = 1/2$, we have central symmetry.

The solution of problem (1)-(3) can be found using the principal solutions (fundamental functions) of the problem and has the form

$$T = \int_0^t d\tau \int_{R_1}^{R_2} E(t - \tau, r, \rho) f_1(\tau, \rho) \rho^{2\nu+1} d\rho + \int_0^t [W_-(t - \tau, r) \omega_1(\tau)$$

$$+ W_+(t - \tau, r) \omega_2(\tau)] d\tau + \frac{\partial}{\partial t} \int_{R_1}^{R_2} K(t, r, \rho) f_2(\rho) \rho^{2\nu+1} d\rho + \int_{R_1}^{R_2} K(t, r, \rho) \left[f_3(\rho) + \frac{b_1^2}{b_0^2} f_2(\rho) \right] \rho^{2\nu+1} d\rho. \tag{4}$$

If on the boundary $r = R_2$ of a cylindrical body $(\nu = 0)$ there is an instantaneous "thermal shock" of power t_0, the temperature fields are given by

$$T = \pi t_0 \sum_{n=1}^{\infty} \frac{I_0(\beta_n R_1) I_0(\beta_n R_2) u_0(\beta_n r, \beta_n R_1)}{I_0^2(\beta_n R_1) - I_0^2(\beta_n R_2)} \left[1 - e^{-kt} \left(\operatorname{ch} \bar{q}_n t + \frac{b_1}{q_n} \operatorname{sh} \bar{q}_n t \right) \right],$$

$$\bar{T} = \pi t_0 \sum_{n=1}^{\infty} \frac{I_0(\beta_n R_1) I_0(\beta_n R_2) u_0(\beta_n r, \beta_n R_1)}{I_0^2(\beta_n R_1) - I_0^2(\beta_n R_2)} (1 - e^{-a^2\beta_n^2 t}). \tag{5}$$

Equations (5) show that as time passes $(t \to \infty)$ the temperature fields become stabilized to a steady-state temperature

$$T_{st} = t_0 \frac{\ln r - \ln R_1}{\ln R_2 - \ln R_1}.$$

LITERATURE CITED

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